

Gupta–Bleuler’s quantization of the anisotropic parity-even and CPT-even electrodynamics of standard model extension

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We have established the Gupta-Bleuler quantization of the photon belonging to the anisotropic parity-even sector of the CPT-even and Lorentz-violating nonbirefringent electrodynamics of the standard model extension. We first present a rule for the Maxwell electrodynamics to be successfully quantized via Gupta-Bleuler technique in the Lorentz gauge. Recognizing the failure of the Gupta-Bleuler method in the Lorentz gauge, $\partial_\mu A^\mu = 0$, for this massless LV theory, we argue that Gupta-Bleuler can be satisfactorily implemented by choosing a modified Lorentz condition, $\partial_\mu A^\mu + \kappa^{\mu\nu} \partial_\mu A_\nu = 0$, where $\kappa^{\mu\nu}$ represents the Lorentz-violation in photon sector. By using a plane-wave expansion for the gauge field, whose polarization vectors are determined by solving an eigenvalue problem, and a weak Gupta-Bleuler condition, we obtain a positive-energy Hamiltonian in terms of annihilation and creation operators. The field commutation relation is written in terms of modified Pauli-Jordan functions, revealing the preservation of microcausality for sufficiently small LV parameters.

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I. INTRODUCTION

The gauge sector of the standard model extension (SME) [1], composed of a CPT-odd [2] and a CPT-even part [3], has also been investigated in connection with Quantum Electrodynamics (QED) [4] and quantization aspects. The CPT-odd part was first addressed, involving the analysis of consistency (causality, stability, and unitarity) as well [5, 6]. Similar consistency analysis about the CPT-even sector, based on the evaluation of the Feynman propagator, was performed for the parity-even and parity-odd sectors in Refs. [7]. The parity-odd nonbirefringent photonic sector was also investigated in Ref. [8], which contains a broader analysis including the basis of polarization vectors, coupling with fermions and evaluation of typical QED processes. An analogue investigation was developed for the birefringent CPT-even gauge sector of the SME [9]. Moreover, to make possible the treatment of infrared divergences that plague the modified QED by the CPT-even gauge sector of the SME, a mass term for photons was supposed [10]. Quantization and consistency aspects were also investigated in the context of Lorentz-violating theories with higher dimension operators [11]. The covariant quantization of the photonic CPT-even sector was addressed in Ref. [12], where it was discussed the quantization in the context of an indefinite metric Hilbert space. It was also reported the necessity of using a weak Lorentz gauge condition in order to correctly implement the Gupta-Bleuler procedure. The covariant quantization of the CPT-even gauge sector of the SME was recently discussed in Refs. [13, 14], in the presence of the Proca mass term in order to avoid some incompatibilities, turning the quantization procedure free from contradictions. In these works, the photon mass has really worked out as a regulator, yielding the definition of polarization basis that allows the consistent quantization of the theory.

The present work is proposed to analyze the canonical quantization of the Maxwell massless electrodynamics modified by the nonbirefringent coefficients κ^{ij} contained in the full CPT-even tensor $(k_F)^{\mu\nu\alpha\beta}$ of the SME. The key point is a modified Lorentz gauge condition, including the LV tensor $\kappa^{\mu\nu}$, which fulfills a general condition for implementing the covariant quantization method for non-massive photons. A plane wave expansion is written in terms of the creation and annihilation operators and polarization vectors. The polarization vectors are achieved as the eigenvalues of an operator defining the equation of motion in this gauge. The Hamiltonian reveals to be positive-definite considering the smallness of the LV parameters. The microcausality is investigated in terms of a generalized Pauli-Jordan function which includes LV contributions. Once the noncovariant quantization was performed, we discuss the Gupta-Bleuler quantization procedure in this theory. Finally, we present our remarks and conclusions.

II. THE CPT-EVEN ELECTROMAGNETIC SECTOR OF SME

The CPT-even photonic sector of the SME described by the minimal Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(k_F)^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}, \quad (1)$$

where the LV background $(k_F)^{\mu\nu\alpha\beta}$ has the same symmetries as the Riemann’s tensor and a null double trace, $(k_F)^{\mu\nu}{}_{\mu\nu} = 0$, providing a total of 19 independent components, from which 10 are birefringent. The terms associated to birefringence are strongly constrained by data of distant galaxies [3], so we can retain only the remaining 9 nonbirefringent, which are parameterized by a symmetric

and traceless tensor $\kappa^{\mu\nu}$ [15] as

$$(k_F)_{\mu\nu\alpha\beta} = \frac{1}{2} (\eta_{\mu\alpha}\kappa_{\nu\beta} - \eta_{\mu\beta}\kappa_{\nu\alpha} + \eta_{\nu\beta}\kappa_{\mu\alpha} - \eta_{\nu\alpha}\kappa_{\mu\beta}), \quad (2)$$

where $\kappa^{\nu\beta} = (k_F)_\mu^{\nu\beta}$. Such parameterization allows to write the Lagrangian density (1) as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\kappa_{\nu\rho}F^{\mu\nu}F_\mu{}^\rho. \quad (3)$$

The equation of motion for the gauge field reads

$$0 = (\Box + \kappa^{\alpha\rho}\partial_\alpha\partial_\rho)A^\beta + \Box\kappa^{\beta\rho}A_\rho - \kappa^{\beta\rho}\partial_\rho\partial_\alpha A^\alpha - \partial^\beta(\partial_\alpha A^\alpha + \kappa^{\alpha\rho}\partial_\alpha A_\rho), \quad (4)$$

with the last term suggesting a generalized gauge fixing condition

$$\partial_\alpha A^\alpha + \kappa^{\alpha\rho}\partial_\alpha A_\rho = 0, \quad (5)$$

in the place of the usual Lorentz condition. Thus, the equation of motion is reduced as

$$(\Box + \kappa^{\alpha\rho}\partial_\alpha\partial_\rho)A^\beta + \Box\kappa^{\beta\rho}A_\rho + \kappa^{\beta\sigma}\kappa^{\alpha\rho}\partial_\sigma\partial_\alpha A_\rho = 0. \quad (6)$$

The parity-even isotropic and anisotropic coefficients are κ_{00} and κ_{ij} , respectively, while κ_{0i} are the parity-odd components. In the next sections we propose a correct implementation of the Gupta-Bleuler quantization by using the gauge condition (5) for the anisotropic parity-even and CPT-even sector of the electrodynamics described by the Lagrangian (3).

III. GUPTA-BLEULER QUANTIZATION

A covariant prescription is accomplished by the Gupta-Bleuler quantization [16, 17], which can become inconsistent in the context of LV theories [13, 14]. This procedure introduces in the Lagrangian density a covariance term breaking the local gauge invariance but not eliminating any gauge field degree of freedom. For instance, in the Maxwell electrodynamics, the Lorentz condition, $(\partial_\mu A^\mu)^2/2\xi$, works well in the Feynman gauge ($\xi = 1$) [18].

The Lagrangian density for Maxwell electrodynamics in Lorentz gauge is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2, \quad (7)$$

which gives the following equation of motion, in momentum space, $O_{\mu\nu}^{(M)}\tilde{A}^\nu(p) = 0$, where we have defined the tensor

$$O_{\mu\nu}^{(M)} = p^2 g_{\mu\nu} - (1 - \xi^{-1})p_\mu p_\nu, \quad (8)$$

whose determinant $-\xi^{-1}(p^2)^4$ provides the dispersion relation $p^2 = 0$. It is easy to verify that for $\xi \neq 1$,

the null-space of the matrix $O_{\mu\nu}^{(M)}|_{p^2=0}$ has dimension 3. On the other hand, for $\xi = 1$, the null-space of the matrix $O_{\mu\nu}^{(M)}|_{p^2=0}$ has dimension 4. The idea following this observation is that a set of four eigenvectors belonging to the null-space of the matrix $O_{\mu\nu}^{(M)}|_{p^2=0}$, for $\xi = 1$, constitute a set of polarization vectors allowing a plane wave expansion for the gauge field. With a well defined set of polarization vectors satisfying the dispersion relation, the implementation of the Gupta-Bleuler quantization follows naturally.

We now use this rule to justify the failure of the Gupta-Bleuler method in the context of the Lorentz-violating electrodynamics defined by Eq. (3), for the Lorentz condition in the Feynman gauge, such as it was recently reported [13, 14]. We start from the Lagrangian density (3) in the presence of the Lorentz gauge term,

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\kappa_{\nu\rho}F^{\mu\nu}F_\mu{}^\rho - \frac{1}{2\xi}(\partial^\mu A_\mu)^2, \quad (9)$$

where $\kappa_{\nu\rho}$ is parameterized as

$$\kappa_{\mu\nu} = \ell(u_\mu v_\nu + u_\nu v_\mu), \quad (10)$$

and u_μ, v_μ , are spacelike four-vectors, $u_\mu = (0, \mathbf{u})$, $v_\mu = (0, \mathbf{v})$, with \mathbf{u} and \mathbf{v} being orthonormal vectors. In the Feynman gauge, $\xi = 1$, the equation of motion of the gauge field reads $O_{\mu\nu}^{(LF)}(p)\tilde{A}^\nu(p) = 0$, where

$$O_{\mu\nu}^{(LF)}(p) = (p^2 + \kappa_{\alpha\beta}p^\alpha p^\beta)g_{\mu\nu} + p^2\kappa_{\mu\nu} - p_\mu\kappa_{\nu\beta}p^\beta - p_\nu\kappa_{\mu\beta}p^\beta. \quad (11)$$

The condition $\det O_{\mu\nu}^{(LF)}(p) = 0$ provides the three dispersion relations: $p^2 = 0$, and

$$\boxplus(p) = p^2 + \kappa^{\alpha\beta}p_\alpha p_\beta = 0, \quad (12)$$

$$\boxtimes(p) = (1 - \ell^2)p^2 + \kappa^{\alpha\beta}p_\alpha p_\beta + (\kappa^2)^{\alpha\beta}p_\alpha p_\beta = 0, \quad (13)$$

where $(\kappa^2)_{\mu\nu} = \kappa_\mu{}^\beta\kappa_{\beta\nu}$. The first one has multiplicity 2, being a nonphysical dispersion relation, while the others have multiplicity 1 and are the physical dispersion relations. It can be verified that the dimension of the null space of $O_{\mu\nu}^{(LF)}|_{p^2=0}$ is 1, i. e., we have only one polarization vector associated with the dispersion relation $p^2 = 0$. The same holds for the other dispersion relations: the dimension of the null space of $O_{\mu\nu}^{(LF)}|_{\boxplus=0}$ and $O_{\mu\nu}^{(LF)}|_{\boxtimes=0}$ are both 1. The null spaces of the tensor (11), evaluated in all dispersion relations, provide a total of three eigenvectors. Hence, the Gupta-Bleuler technique cannot be applied satisfactorily in this Lorentz-violating electrodynamics, for the usual Lorentz condition in the Feynman gauge.

We solve this problem by selecting the gauge condition (5) which represents a Lorentz-violating generalization

of the Lorentz condition. It will reveal to be a suitable choice to successfully implement the Gupta-Bleuler quantization in this model. Hence, the Lagrangian density (3) becomes

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\kappa_{\nu\rho}F^{\mu\nu}F_{\mu}{}^{\rho} - \frac{1}{2\xi}(\partial_{\mu}A^{\mu} + \kappa^{\mu\nu}\partial_{\mu}A_{\nu})^2. \quad (14)$$

By working in Feynman gauge, $\xi = 1$, the equation of motion in the momentum space for the gauge field reads

$$\mathcal{O}_{\mu\nu}(p)\tilde{A}^{\nu}(p) = 0, \quad (15)$$

with $\mathcal{O}_{\mu\nu}$ is defined by

$$\mathcal{O}_{\mu\nu}(p) = g_{\mu\nu}(p^2 + \kappa^{\alpha\beta}p_{\alpha}p_{\beta}) + \kappa_{\mu\nu}p^2 + \kappa_{\mu\alpha}\kappa_{\beta\nu}p^{\alpha}p^{\beta}, \quad (16)$$

whose determinant is $-\left[\boxplus(p)\right]^3\boxtimes(p)$. Here, we also identify the two physical dispersion relations given in Eqs. (12) and (13). The null space of $\mathcal{O}_{\mu\nu}$, when the dispersion relation $\boxtimes(p) = 0$ is satisfied, is 1 providing one polarization vector. On the other hand, the dimension of the null space of $\mathcal{O}_{\mu\nu}|_{\boxplus(p)=0}$ is 3, i. e., there are three polarization vectors associated with the dispersion relation $\boxplus(p) = 0$, one of them becomes physical and the other two are not. So, the gauge condition (5), with $\xi = 1$, provides 4 polarizations vectors associated with the dispersion relations, fulfilling the condition for implementing the Gupta-Bleuler quantization.

The canonical conjugate momentum is

$$\pi^{\mu} = -(g^{\mu\nu} + \kappa^{\mu\nu})\dot{A}_{\nu}, \quad (17)$$

which allows to impose the following canonical commutation relations

$$[A_{\mu}(t, \mathbf{x}), \pi^{\nu}(t, \mathbf{y})] = i\delta_{\mu}{}^{\nu}\delta^3(\mathbf{x} - \mathbf{y}), \quad (18)$$

whereas the other are null.

The modified Lorentz condition, $\partial_{\mu}(A^{\mu} + \kappa^{\mu\nu}A_{\nu}) = 0$, allows to define four polarization vectors which can be used to propose a solution for Eq. (15) as a plane-wave expansion,

$$A_{\mu}(x) = \sum_{\lambda=0}^3 \int \widehat{d^3\mathbf{p}}^{(\lambda)} \left[a_{(\lambda)}(\mathbf{p}) e^{-ix \cdot p^{(\lambda)}} + h.c. \right] \varepsilon_{\mu}^{(\lambda)}(\mathbf{p}), \quad (19)$$

where $\widehat{d^3\mathbf{p}}^{(\lambda)} = d^3\mathbf{p}/\sqrt{(2\pi)^3 2E^{(\lambda)}}$, the annihilation and creation operators are described by $a_{(\lambda)}(\mathbf{p})$ and $a_{(\lambda)}^{\dagger}(\mathbf{p})$, respectively. The polarization vectors $\varepsilon_{\mu}^{(\lambda)}(\mathbf{p})$ satisfy the following eigenvalue equation:

$$\mathcal{O}^{\mu\nu}\varepsilon_{\nu}^{(\lambda)} = \alpha^{(\lambda)}(g^{\mu\nu} + \kappa^{\mu\nu})\varepsilon_{\nu}^{(\lambda)}. \quad (20)$$

The eigenvalues $\alpha^{(\lambda)}$ are

$$\alpha^{(0)} = \alpha^{(1)} = \alpha^{(2)} = p^2 + \kappa_{\mu\nu}p^{\mu}p^{\nu}, \quad (21)$$

$$\alpha^{(3)} = p^2 + \frac{\kappa_{\mu\nu}p^{\mu}p^{\nu} + (\kappa^2)_{\mu\nu}p^{\mu}p^{\nu}}{1 - \ell^2}. \quad (22)$$

yielding the four polarization vectors

$$\varepsilon_{\mu}^{(0)} = (1, \mathbf{0}), \quad \varepsilon_{\mu}^{(1)} = (0, \varepsilon_i^{(1)}), \quad (23)$$

$$\varepsilon_{\mu}^{(2)} = (0, \varepsilon_i^{(2)}) \quad , \quad \varepsilon_{\mu}^{(3)} = (0, \varepsilon_i^{(3)}), \quad (24)$$

with

$$\varepsilon_i^{(1)} = n^{(1)}p_i \quad , \quad \varepsilon_i^{(3)} = n^{(3)}\left(d_{(2)}^{-1}\right)_{ij}\epsilon_{jka}p_kw_a, \quad (25)$$

$$\varepsilon_i^{(2)} = n^{(2)}\left[(d_{(2)})_{ab}p_ap_bw_i - (p_aw_a)p_i\right], \quad (26)$$

where $w_i = \epsilon_{ijk}u_jv_k$, and $n^{(i)}$ are normalization constants. The polarization vectors satisfy the normalization condition given by $\varepsilon_{\mu}^{(\lambda)}(g^{\mu\nu} + \kappa^{\mu\nu})\varepsilon_{\nu}^{(\lambda')} = g^{\lambda\lambda'}$, while the completeness relation reads

$$\sum_{\lambda=0}^3 g^{\lambda\lambda'}\varepsilon_{\mu}^{(\lambda)}\varepsilon_{\nu}^{(\lambda')} = (g + \kappa)_{\mu\nu}^{-1}. \quad (27)$$

The energy for each polarization vector is

$$E^{(0)} = E^{(1)} = E^{(2)} = |\mathbf{p}|\sqrt{1 - \kappa_{ij}p_ip_j/p^2}, \quad (28)$$

$$E^{(3)} = |\mathbf{p}|\sqrt{1 - \frac{\kappa_{ij}p_ip_j + (\kappa^2)_{ij}p_ip_j}{(1 - \ell^2)p^2}}, \quad (29)$$

which coincides with the dispersion relations obtained in Ref. [7]. The energies are real numbers whenever the Lorentz-violating parameters κ_{ij} are sufficiently small. In order to satisfy the canonical commutation relation (18), the creation and annihilation operators must satisfy the standard commutation relations,

$$[a_{(\lambda)}^{\dagger}(p), a_{(\lambda')}(q)] = g_{\lambda\lambda'}\delta^3(\mathbf{p} - \mathbf{q}), \quad (30)$$

and the others being null. The goal of our choice for the polarization (24) is to express the quantum Hamiltonian as an explicit sum of the contributions of each polarization mode, as required,

$$H = -\sum_{\lambda=0}^3 \int d^3\mathbf{p} g_{\lambda\lambda} E^{(\lambda)} N^{(\lambda)}, \quad (31)$$

where $N^{(\lambda)} = a_{(\lambda)}^{\dagger}a_{(\lambda)}$ is the number operator counting (λ) mode. Despite the Hamiltonian can be expressed in a simple form, it is not positive-definite. Moreover, at operator level, the gauge condition $\partial_{\mu}A^{\mu} + \kappa^{\mu\nu}\partial_{\mu}A_{\nu} = 0$ is not compatible with the commutation relation (18). From Eq. (30), we observe that the temporal creation and annihilation operators, $a_{(0)}^{\dagger}$ and $a_{(0)}$, satisfy a commutation relation with changed signal, providing states with negative norm. These problems can be solved in a covariant way by imposing the Gupta-Bleuler condition[16, 17], i.e.,

the physical states $|\varphi\rangle$ are those providing null expectation value for the modified gauge condition,

$$\langle\varphi|(g^{\mu\nu} + \kappa^{\mu\nu})\partial_\mu A_\nu|\varphi\rangle = 0. \quad (32)$$

This is a strong operator condition. The physical states can be selected imposing a weaker operator condition,

$$(g^{\mu\nu} + \kappa^{\mu\nu})\partial_\mu A_\nu^{(+)}|\varphi\rangle = 0, \quad (33)$$

where the gauge field was decomposed in positive and negative frequencies, $A_\mu = A_\mu^{(+)} + A_\mu^{(-)}$, respectively. We now explicitly implement the weak condition (33) in the plane-wave expansion (19) of the gauge field, attaining

$$\sum_{\lambda=0}^3 \int d^3\mathbf{p}^{(\lambda)} e^{-ix \cdot p^{(\lambda)}} \left[p_\mu^{(\lambda)} (g^{\mu\nu} + \kappa^{\mu\nu}) \varepsilon_\nu^{(\lambda)} \right] a_{(\lambda)} |\varphi\rangle = 0. \quad (34)$$

It is verified that contributions coming from polarizations $\varepsilon_\nu^{(2)}$ and $\varepsilon_\nu^{(3)}$ are null. The remaining polarizations yield $p_\mu^{(\lambda)} (g^{\mu\nu} + \kappa^{\mu\nu}) \varepsilon_\nu^{(\lambda)} = (-1)^\lambda E^{(\lambda)}$, for $\lambda = 0, 1$, with $E^{(1)} = E^{(0)}$, which allows to achieve the following constraint for the physical states:

$$[a_{(0)} - a_{(1)}] |\varphi\rangle = 0. \quad (35)$$

The last expression links the expectation value of the scalar and longitudinal photon operator numbers,

$$\langle\varphi|N^{(0)}|\varphi\rangle = \langle\varphi|N^{(1)}|\varphi\rangle, \quad (36)$$

a supplementary condition (36) which solves the problem concerning the negative energy contributions in the Hamiltonian (31). Once we have successfully quantized this Lorentz-violating electrodynamics, we can also compute the covariant gauge field commutation relation

$$[A_\mu(x), A_\nu(y)] = \int \sum_{\lambda=0}^3 \frac{d^3\mathbf{p}}{(2\pi)^3 2E^{(\lambda)}} \times \left(e^{i(x-y) \cdot p^{(\lambda)}} - e^{-i(x-y) \cdot p^{(\lambda)}} \right) g_{\lambda\lambda} T_{\mu\nu}^{(\lambda)}, \quad (37)$$

where $T_{\mu\nu}^{(\lambda)} = \varepsilon_\mu^{(\lambda)}(p) \varepsilon_\nu^{(\lambda)}(p)$. Using the dispersion relations and the completeness relation (27), the result is

$$[A_\mu(x), A_\nu(y)] = T_{\mu\nu}^{(3)}(i\partial) i\Delta^{(3)}(x-y) - \left[(g + \kappa)_{\mu\nu}^{-1} + T_{\mu\nu}^{(3)}(i\partial) \right] i\Delta^{(2)}(x-y), \quad (38)$$

with $\Delta^{(2)}(x-y)$ and $\Delta^{(3)}(x-y)$ being the generalized Pauli Jordan functions defined by

$$\Delta^{(\beta)}(x_0, \mathbf{x}) = -\frac{\varepsilon(x_0) \delta\left((x_0)^2 - \left(d_{(\beta)}^{-1}\right)_{ij} x_i x_j\right)}{2\pi \sqrt{\det(d_{(\beta)})}}, \quad (39)$$

with

$$(d_{(2)})_{ij} = \delta_{ij} - \kappa_{ij}, \quad \left(d_{(3)}^{-1}\right)_{ij} = \delta_{ij} + \kappa_{ij}. \quad (40)$$

The argument of the δ -function in (39) preserves the light-cone structure (deformed or not) whenever $\left(d_{(\beta)}^{-1}\right)_{ij} x_i x_j > 0$. Such requirement is guaranteed if the matrix $\left(d_{(\beta)}^{-1}\right)$ is positive-definite, which holds since the Lorentz-violating parameters κ_{ij} are sufficiently small (the same condition that yields energy positivity). This way, the function $\Delta^{(\beta)}(x)$ vanishes for spacelike vectors implying microcausality is assured.

IV. FINAL REMARKS

We conclude that we have successfully implemented the Gupta-Bleuler method for this anisotropic parity-even and CPT-even electrodynamics without recourse to an infrared regularization or to a small mass for the photon field, as already known in the literature. Our results can also be verified via the Dirac's quantization method for constrained systems. This formalism shows that in this LV electrodynamics, the temporal gauge ($A_0 = 0$) is not compatible with the usual Coulomb gauge ($\partial_j A_j = 0$), but with a modified LV version, $(\delta_{ij} - \kappa_{ij})\partial_i A_j = 0$, as it will be reported elsewhere [19].

We believe that the covariant quantization formalism here derived may be applied to other coefficients of the CPT-even tensor, as the parity-odd ones, as far as in the context of other electrodynamic models endowed with anisotropic or higher dimension terms.

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